

Meeting of the US Nuclear Data Network  
Brookhaven National Laboratory  
National Nuclear Data Center  
April 16-17, 2001

Training Sessions  
Topics for Discussion

1. DDEP Evaluations into ENSDF (Helmer)  
General Policies.  
Conversion Coefficients and Derived Quantities
2. Half-life (Helmer)  
Discrepant data  
Various Types of Averages
3.  $\beta^-$  and  $\beta^+$  Decay (Browne)  
Level Feedings. Transition Intensity Balances. The GTOL program.  
Annihilation Radiation.  $\beta^+$  Branching. The LOGFT program.
4. Electron Capture Decay (Browne)  
EC/ $\beta^+$  ratios  
X Rays. The RADLST program.  
The use of X-ray intensities for verifying decay-scheme consistency.
5. Gamma Rays (Browne, Helmer)  
Energies. New Evaluated Standards (Helmer)  
Intensities. Averaging. The LWEIGHT program. (Browne)  
Multipolarities. Mixing Ratios. (Browne)  
Decay-Scheme Normalization. Examples. (Browne)  
Absolute Intensities. Uncertainties. The GABS program (Browne)  
Conversion Coefficients. Theoretical Values. Current Status. (Browne)

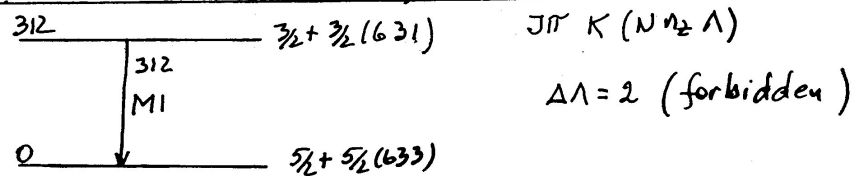
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April 16-17, 2001

## Theoretical Conversion Coefficients

1. F. Rösler et al., At. Data and Nucl. Data Tables 21, 92 (1978)  
 $Z \geq 30$
2. I. M. Baul et al., At. Data and Nucl. Data Tables 18, 433 (1976)  
 $Z < 30$

Interpolation Program ICC (Yakushev and Coursol).

## Nuclear Penetration Effects



The conversion coefficient is:

$$\beta = \beta_0 (1 + B_1 \lambda + B_2 \lambda^2)$$

$\beta_0$  is the conventional conv. coeff.

$B_1$  and  $B_2$  are parameters tabulated in Nucl. Data Tables A6, 1 (1969).

$\lambda = \frac{\langle J' | P | J \rangle}{\langle J' | M_1 | J \rangle}$  is the penetration parameter.

$\langle J' | P | J \rangle$  is the  $M1$  Penetration Matrix Element

$\langle J' | M_1 | J \rangle$  is the Reduced Matrix Element for  $\gamma$ -ray emission.

## More about multipolarities

$$\begin{array}{c}
 0^+ \\
 \hline
 \downarrow \text{E0} \\
 0^+
 \end{array}
 \quad |J_i - J_f| \leq l \leq |J_i + J_f|; \quad 0 \leq l \leq 0; \quad l=0$$

$$\begin{array}{c}
 2^+ \\
 \hline
 \downarrow \text{E0} + \text{M1} + \text{E2} \\
 2^+
 \end{array}
 \quad |2-2| \leq l \leq |2+2|; \quad 0 \leq l \leq 4; \quad l=0, 1, 2,$$

$$\begin{array}{c}
 \frac{1}{2}^+ \\
 \hline
 \downarrow \text{E0} + \text{M1} \\
 \frac{1}{2}^+
 \end{array}
 \quad |\frac{1}{2} - \frac{1}{2}| \leq l \leq |\frac{1}{2} + \frac{1}{2}|; \quad 0 \leq l \leq 1; \quad l=0, 1$$

$$\begin{array}{c}
 1^+ \\
 \hline
 \downarrow \text{M1} \\
 0^+
 \end{array}
 \quad |1-0| \leq l \leq |1+0|; \quad 1 \leq l \leq 1; \quad l=1$$

$$\begin{array}{c}
 \frac{3}{2}^+ \\
 \hline
 \downarrow \text{M1} + \text{E2} \\
 \frac{1}{2}^+
 \end{array}
 \quad |\frac{3}{2} - \frac{1}{2}| \leq l \leq |\frac{3}{2} + \frac{1}{2}|; \quad 1 \leq l \leq 2; \quad l=1, 2$$

### Conventions

- M1 - Definite M1 mult.
- (M1) - Uncertain M1 mult.
- M1(E2) - Definite M1 with possible E2 mixing.
- [M1] - Assumed M1 mult.

## Multipolarities and Mixing Ratios

1. From conversion-electron data.

2. From  $\delta(\theta)$

$$\text{Mixing Ratio} \Rightarrow \delta^2 = \frac{\langle E2 \rangle^2 E_\gamma^5}{\langle M1 \rangle^2 E_\gamma^3} = \frac{TP(E2)}{TP(M1)}$$

Competition: M1 E2 M3 E4 ... (rotational E2)  
E1 M2 E3 M4 ... (delayed level,

$$TP = TP(E2) + TP(M1) = (1 + \delta^2) TP(M1)$$

$$\therefore \begin{aligned} TP(M1) &= \frac{1}{1 + \delta^2} TP & (\delta = 0 \rightarrow \text{PURE } M1) \\ TP(E2) &= \frac{\delta^2}{1 + \delta^2} TP & (\delta = \infty \rightarrow \text{PURE } E2) \end{aligned}$$

Case 1. We measured  $\alpha(\text{exp})$ . Deduce  $\delta^2$ .

$$I_e = I_\gamma(M1) \alpha^{th}(M1) + I_\gamma(E2) \alpha^{th}(E2)$$

$$\alpha(\text{exp}) = \frac{I_e}{I_\gamma} = \frac{I_\gamma(M1)}{I_\gamma} \alpha^{th}(M1) + \frac{I_\gamma(E2)}{I_\gamma} \alpha^{th}(E2)$$

$$\text{But } I_\gamma(M1) = \frac{1}{1 + \delta^2} I_\gamma \text{ and } I_\gamma(E2) = \frac{\delta^2}{1 + \delta^2} I_\gamma$$

$$\therefore \alpha(\text{exp}) = \frac{1}{1 + \delta^2} (\alpha^{th}(M1) + \delta^2 \alpha^{th}(E2))$$

$$\text{or } \boxed{\delta^2 = \frac{\alpha^{th}(M1) - \alpha(\text{exp})}{\alpha(\text{exp}) - \alpha^{th}(E2)}}$$

Case 2. We measured  $R = \frac{I_{L1}}{I_{L3}}$ . Deduce  $\delta^2$ .

$$I_{L1} = I_Y(M1) \alpha_{L1}^{th}(M1) + I_Y(E2) \alpha_{L1}^{th}(E2)$$

$$I_{L3} = I_Y(M1) \alpha_{L3}^{th}(M1) + I_Y(E2) \alpha_{L3}^{th}(E2)$$

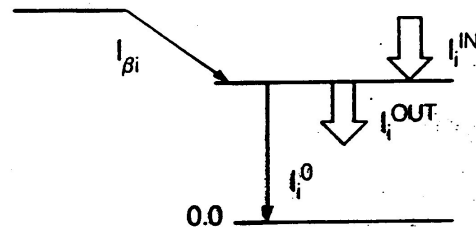
$$\text{Also } I_Y(E2) = \frac{\delta^2}{1+\delta^2} I_Y ; I_Y(M1) = \frac{1}{1+\delta^2} I_Y$$

$\therefore$

$$\frac{\delta^2}{1+\delta^2} = \frac{R \alpha_{L3}^{th}(M1) - \alpha_{L1}^{th}(M1)}{\alpha_{L1}^{th}(E2) - \alpha_{L1}^{th}(M1) + R [\alpha_{L3}^{th}(M1) - \alpha_{L3}^{th}(E2)]}$$

$$\% E2 \text{ is } 100 \times \frac{\delta^2}{1+\delta^2} ; \% M1 = 100 \times \frac{1}{1+\delta^2}$$

# x-ray intensity balance



The corresponding normalizing factor is

$$N = \frac{100}{\sum_i \Delta_i} = \frac{100}{D + \sum_i I_i^0}$$

$$\Delta_i = I_i^{OUT} + I_i^0 - I_i^{IN}$$

$$D = \sum_i (I_i^{OUT} - I_i^{IN}) \equiv 0 \quad \text{ALWAYS !!}$$





## 2. Using X-ray intensity to normalize a decay scheme

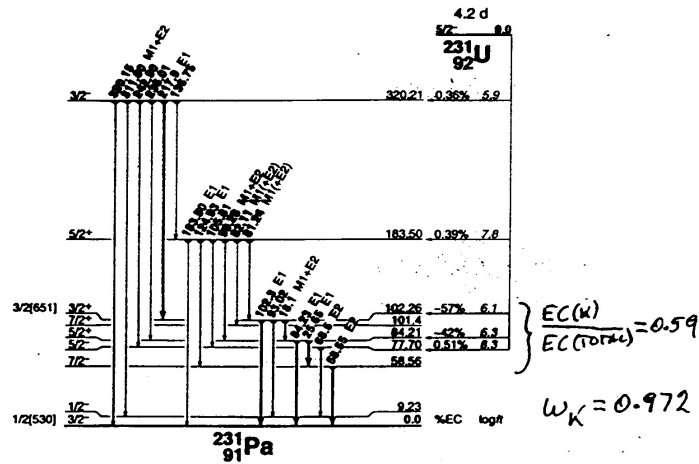


Fig. 4.  $^{231}\text{U}$  electron-capture decay scheme. Gamma rays measured in this work are shown with thicker arrows; other data are from refs. [3,11]. Electron-capture branches per 100 decays of  $^{231}\text{U}$  and log ft values are from gamma-ray transition probability balances (see Table 3).

$$I_{\gamma}(25) = 100 (6) ; I_{\gamma}(84) = 50 (3) ; I_{KX} = 390 (14)$$

$$B_K = 115.6 \text{ keV (most KX rays originated from atomic vacancies created by the EC process)}$$

$$\text{Total number of vacancies} = \frac{I_{KX}}{w_K \cdot \frac{EC(K)}{EC(TOTAL)}} = 680 (33)$$

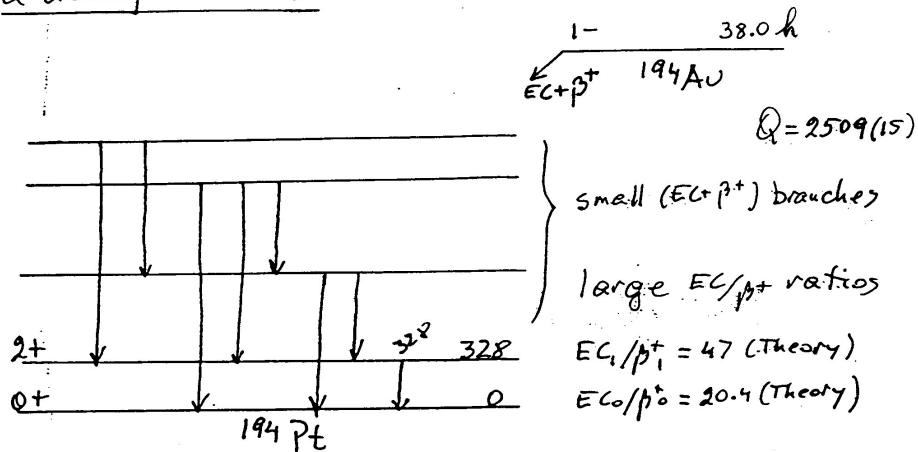
$$N = \frac{100}{680 (33)} = 0.147 (7)$$

So:

$$I_{\gamma}(25) = 100 (6) \times 0.147 (7) = 15 (1) \%$$

$$I_{\gamma}(84) = 50 (3) \times 0.147 (7) = 7.5 (6) \%$$

3. Using annihilation radiation to normalize a decay scheme.



All  $I_\gamma$ 's are on a relative scale ( $I_\gamma(328) = 100$ )

Measured  $I(511 \gamma^\pm)/I_\gamma(328) = 0.058(4)$

So  $I(\beta^+, \text{total}) = \frac{1}{2} \times 100 \times 0.058(4) = 2.9(2)$

$\beta^+$  decay populates the first two levels (approximation,  
 $I(\beta^+_0) + I(\beta^+_1) = 2.9(2)$ )

From  $\gamma$ -ray intensity balance:  $I(\text{EC}_1) + I(\beta^+_1) = 49(5)$

Using theory:  $\text{EC}_1/\beta^+_1 = 47 \rightarrow I(\beta^+_1) = \frac{49}{48} = 1.02$

Therefore,  $I(\beta^+_0) = 2.9(2) - 1.02 = 1.9$ ,

Using theory,  $\text{EC}_0/\beta^+_0 = 20.4 \rightarrow I(\text{EC}_0) = 38.8$

$\text{EC} + \beta^+$  branch to ground state =  $38.8 + 1.9 = 40.7$

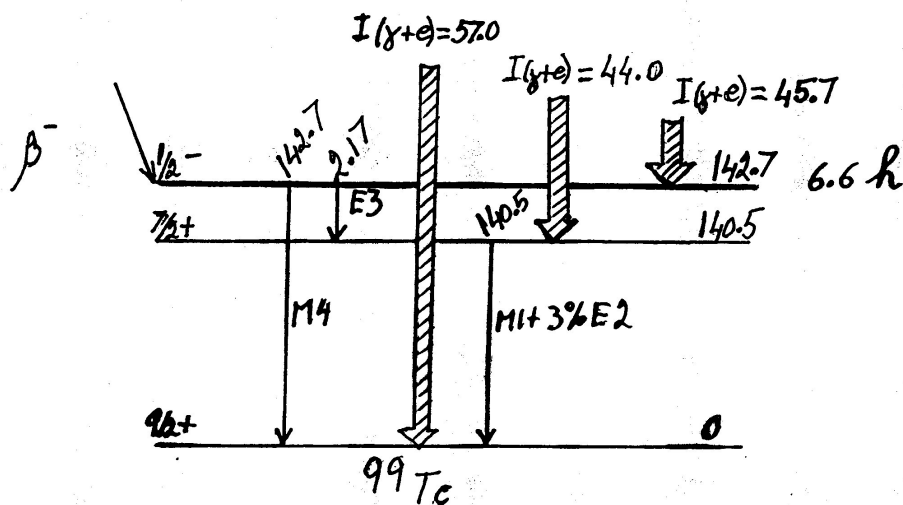
From  $\gamma$ -ray intensity balance:

$(\sum_i I_\gamma(H\alpha_i) + 40.7)N = 100\% \rightarrow N = \frac{100}{162.7} = 0.61$

123

$I_\gamma(328) = 100 \times 0.61 = 61\%$

$$\frac{1/2+}{99M_0} \quad 0 \quad 65.9h$$

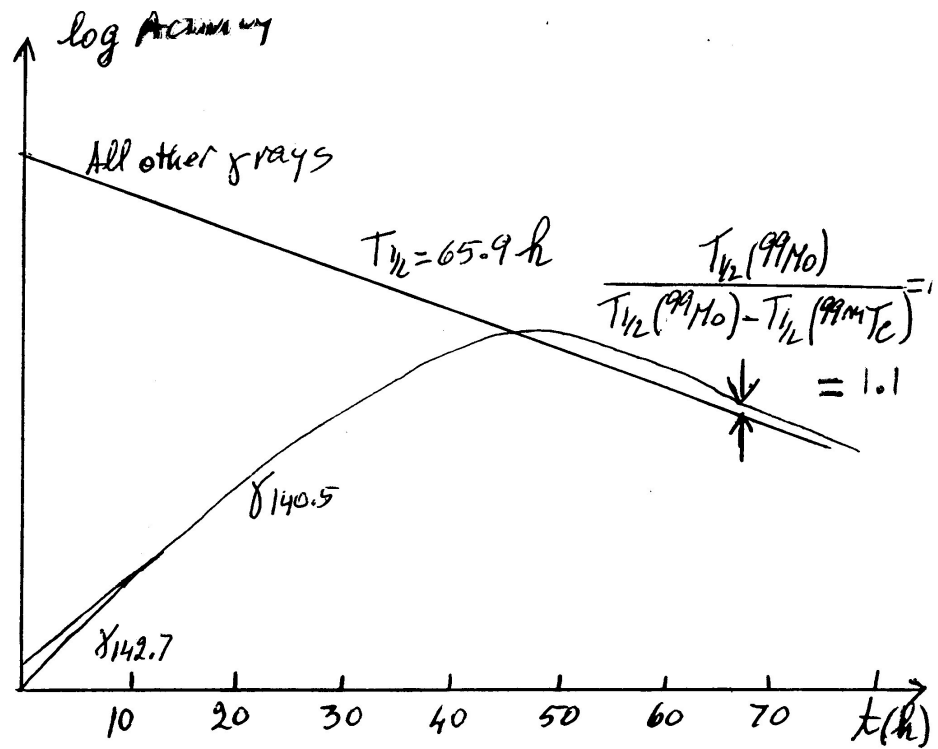


### Equilibrium Intensities

$$I(g+e)(140.5) = 827(12)$$

$$I(g+e)(142.7) = 7.3(7)$$

$$F = \frac{T_{1/2}(99M_0)}{T_{1/2}(99M_0) - T_{1/2}(99mTc)} = 1.10$$



## Decay Scheme Normalization

$$\left[ I_{\gamma(e)}^*(142.7) + I_{\gamma(e)}^*(140.5) + I_{\gamma(e)gs} \right] \times N = 100$$

$$I_{\gamma(e)}^*(142.7) = 7.3(7) / F = 7.3(7) / 1.1 = 6.6(6)$$

$$I_{\gamma(e)}^*(140.5) = 827(12) / F = 827(12) / 1.1 = 752(11)$$

$$I_{\gamma(e)gs} = 57.0(8)$$

$$N = \frac{100}{816(11)} = 0.1226(17)$$

$$P_{\gamma}(140.5) = 739(11) \times 0.1226(16) = 90.6\%$$

$$I_{\gamma}(739.5) = 100 \rightarrow P_{\gamma}(739.5) = 12.26(17)\%$$

What is the uncertainty in  $P_{\gamma}(140.5)$ ?

Is it  $P_{\gamma}(140.5) = 90.6(18)$ ? That is, 2%?

$$\begin{aligned} P_{\gamma}(140.5) &= \frac{I_{\gamma}(140.5) \times 100}{\frac{1}{1.1} \left[ I_{\gamma}(140.5)(1 + \alpha_{140.5}) + I_{\gamma}(142)(1 + \alpha_{142}) \right] + I_{\gamma(e)gs}} \\ &= 100 (1.017(3) + 0.0099(9) + 0.077(2))^{-1} \\ &= 90.6(3)\% \end{aligned}$$

$\beta^-$  Feeding to 142.7-keV level

$$I_{\beta^-} = I_{\gamma_{142.7}}^* (1 + \alpha_{142.7}) + I_{\gamma_{2.17}}^* (1 + \alpha_{2.17}) - I_{\gamma(\gamma^+e)}_{142.7}$$

$$\begin{aligned} I_{\gamma_{2.17}}^* (1 + \alpha_{2.17}) &= I_{\gamma_{140.5}}^* (1 + \alpha_{140.5}) / 1.1 - I_{\gamma(\gamma^+e)}_{140.5} \\ &= \frac{739(11) \times 1.119(13)}{1.1} - 44.0 = 708 \end{aligned}$$

So,

$$I_{\beta^-} = \frac{0.174(14) \times 41.9(8)}{1.1} + 708 - 45.7 = 669$$

$$P_{\beta^-} = 669 \times 0.1226 = 82.0\%$$

\* Corrected for equilibrium

REPORT FILE - PROGRAM GAOS

Current date: 03/22/2001

99MO B- DECAY 1992GO22,1990ME15  
NR= 0.1226 18 BR= 1.00

FOR INTENSITY UNCERTAINTIES OF GAMMA RAYS NOT USED IN CALCULATING NR,  
COMBINE THE UNCERTAINTY IN THE RELATIVE INTENSITY IN QUADRATURE  
WITH THE UNCERTAINTY IN THE NORMALIZING FACTOR (NR x BR).  
FOR THE FOLLOWING GAMMA RAYS:

E= 140.511 1 %IG=82.4 4 PER 100 DIS.(Compare with 82.4 17)  
E= 142.675 25 %IG=0.0194 18 PER 100 DIS.(Compare with 0.0194 18)  
E= 181 %IG=6.08 12 PER 100 DIS.(Compare with 6.08 13)  
E= 761.77 8 %IG=0.0023 14 PER 100 DIS.(Compare with 0.0023 14)

### Absolute Equilibrium Intensities

$$P_{\gamma(140.5)} = 82.4(4) \times 1.1 = 90.6(4) \%$$

$$P_{\gamma(142.7)} = 0.0194(18) \times 1.1 = 0.0213(20) \%$$